Solving Equations: Find x!

Goal is *always* to get *variable* = *constant*

General Strategy
1) If there are parenthesis in the equation, distribute first
2) Combine like terms on each side of the equation
3) Add or subtract terms to get variable terms on one side and constant terms on the other
4) Multiply or divide to isolate the variable

**Solving Equations of the form** $x + a = b$
**Strategy:**
1) To remove a term from one side of the equation, add the opposite of that term to each side of the equation.

*Example:*

*Solution:*
To get x by itself, we want to remove -2 from the left side of the equation
1) Add the opposite of -2, +2 to both sides
$x - 2 + 2 = 7 + 2$

**Solving Equations of the form** $ax = b$
**Strategy:**
1) If the coefficient is an integer, to get the variable by itself (i.e. make the coefficient of the variable equal to 1, divide both sides by the coefficient
2) If the coefficient is a fraction, to get the variable by itself (i.e. make the coefficient of the variable equal to 1, multiply both sides by the reciprocal.

*Example 1:*

*Solution:*
To get x by itself, we want to divide by 5 on both sides of the equation
$\frac{5x}{5} = \frac{10}{5}$
$x = 2$

*Example 2:*

*Solution:*
To get x by itself, we want to multiply the reciprocal $\frac{3}{2}$ on both sides of the equation.
$\frac{3 \cdot \frac{2}{3}}{x} = \frac{14 \cdot \frac{3}{2}}{1}$
$x = 21$
**Solving Equations of the form ax + b = c**

**Strategy:**

1) If your problem contains fractions, multiply the entire equation by the LCD to make your life simpler.

2) Then, remove the constant from one side of the equation by adding the opposite to each side of the equation.

3) Last step is to divide both sides (or multiply) by the coefficient with the variable.

**Example 1:**

_Solve:_ \( 8a - 2 = 13 \)

**Solution:**

First get all the constants to one side

Add the opposite of -2, \(+2\) to both sides

\[ 8a - 2 + 2 = 13 + 2 \]

\[ 8a = 13 + 2 \]

\[ 8a = 15 \]

To get a by itself, we want to divide by 8 on both sides of the equation

\[ \frac{8a}{8} = \frac{15}{8} \]

\[ a = \frac{15}{8} \]

**Example 2:**

_Solve:_ \( \frac{2}{5}x - \frac{1}{4} = \frac{3}{2} \)

**Solution:**

If your problem contains fractions, multiply the entire equation by the LCD to make your life simpler

LCD = 20, so multiply both sides of the equation by 20

\[ 20 \left( \frac{2}{5}x - \frac{1}{4} \right) = \frac{3}{2} \cdot 20 \]

\[ 8x - 5 = 30 \]

Get all the constants to one side

\[ 8x - 5 + 5 = 30 + 5 \]

\[ 8x = 35 \]

Divide both sides by 8

\[ \frac{8x}{8} = \frac{35}{8} \]

\[ x = \frac{35}{8} \]
Solving Equations with variables and constants on both sides of the equation

Strategy:
1) First, combine like terms on each side of the equation
2) Use addition or subtraction to move all variable terms to one side, and constants to the other side
3) Divide or multiply to get variable by itself

Example 1:

*Solve:* \(3n - 2 + 7n = 13 - n + 7\)

Solution:

First simplify as much as possible on each side of the = sign by combining like terms

\(10n - 2 = 20 - n\)

Use addition or subtraction to move variable terms to one side and constants to the other side

\(10n - 2 + 2 + n = 20 - n + 2 + n\)

\(11n = 22\)

Divide both sides of the equation by 11

\[
\frac{11n}{11} = \frac{22}{11}
\]

\(n = 2\)

Solving Equations containing parentheses

Strategy:
1) First distribute to eliminate the parentheses
2) Combine like terms on each side of the equation
3) Use addition or subtraction to move all variable terms to one side, and constants to the other side
4) Divide or multiply to get variable by itself

Example 1:

*Solve:* \(3 + 2[4x - 3(5 - x)] = 3(x - 20)\)

Solution:

Distribute to eliminate parentheses

\(3 + 2[4x - 3(5 - x)] = 3(x - 20)\)
\(3 + 2[4x - 15 + 3x] = 3(x - 20)\)
\(3 + 2[7x - 15] = 3x - 60\)
\(3 + 14x - 30 = 3x - 60\)

Combine like terms

\(14x - 27 = 3x - 60\)

Use addition or subtraction to move variable terms to one side and constants to the other side

\(14x - 27 + 27 = 3x - 60 + 27\)
\(14x - 3x = 3x - 3x - 33\)
\(11x = 33\)

Divide both sides of the equation by 11

\[
\frac{11x}{11} = \frac{33}{11}
\]

\(n = 3\)