The level of confidence, $c$, is the area under the standard normal curve between critical values $-Zc$ and $+Zc$. The area outside the critical values is $1-c$, therefore the area in each tail is $\frac{1}{2}(1-c)$.

A **confidence interval** for the population proportion $p$ is a range of values we expect to contain that unknown population proportion ($\hat{p} \pm$ margin of error).

A **confidence level** is the **probability** that the confidence interval does include the population proportion. The most common confidence level is 95% (i.e., for 95% of random samples of $x$, all of these samples will fall $\pm$ two S.E.s (standard errors) from $p$.

### Assumptions and Conditions

- Independence assumption
- Randomization condition
- 10% condition
- Sample size assumption
- Success/failure condition - the sample must be large enough to accommodate at least 10 successes and 10 failures ($n*p, n*q$)

### Example 1: finding the margin of error.

In a recent poll, 1000 adults employed full-time were asked whether they would be willing to carpool to work if a program was available through their employer. Find the margin of error for this poll if we want 90% confidence in our estimate of the percent of adults who are employed full time and interested in carpool programs.

**Given:**

- $n = 1000$
- $\hat{p} = .25$
Answer:

\[ S.E. (\hat{p}) = \sqrt{\frac{(p-\hat{p})(q-\hat{p})}{n}} = \sqrt{\frac{(.25)(.75)}{1000}} = .014 \]

90% Confidence Interval critical value: 1.645 (This can be verified on the TI-84 by typing \(2^{nd}\) \(\text{Dist}\) \(\text{InvNorm}(.05) = -1.645\); therefore, the endpoints of the confidence interval fall at \(z = -1.645\) and 1.645).

**M.E. (margin of error) = (z-score)(S.E.)**

\[ M.E. = (1.645)(0.014) = \pm .023 \]

**Example 2:** Finding a confidence interval.

1000 customers at a retail grocery store were surveyed and asked the following question: “Do you feel that eating organic foods is important to maintaining good health?” Thirty five percent of 522 people who responded said they felt it was important.

Construct a 95% confidence interval for the true proportion of adults who feel that eating organic foods is important to maintaining good health.

Answer:

\[ n = 522 \]

\[ \hat{p} = .35 \]

95% confidence critical value = \(\pm 1.95\)

\[ S.E. (\hat{p}) = \sqrt{\frac{(p-\hat{p})(q-\hat{p})}{n}} = \sqrt{\frac{(0.35)(0.65)}{522}} = .02 \]

**M.E. = (z-score)(S.E.) = (1.95)(.02) = .039**

\[ \hat{p} \pm M.E. = (.311, .389) \]

**Using TI-84**

To calculate confidence interval for a population proportion: STATS \(\rightarrow\) TESTS \(\rightarrow\) Select A: 1-PropZInt

\[
\begin{array}{c}
1-\text{PropZInt} \\
x = (.35)(522) = 183 \text{ (enter a whole number for } x) \\
n = 522 \\
C-\text{Level} = .95 \\
\text{Calculate}
\end{array}
\]

\[
\begin{array}{c}
\text{1-PropZInt} \\
x = (.35)(522) = 183 \text{ (enter a whole number for } x) \\
n = 522 \\
C-\text{Level} = .95 \\
\text{Calculate}
\end{array}
\]
Note the slight difference between the calculator and non-calculator answers. This is normal to see and results from rounding digits.

Example 3: Constructing a Confidence Interval.

Construct the 95% confidence interval for the population proportion, \( p \), when using the sample statistics below:

\[
\hat{p} = 0.56 \\
\hat{q} = 0.44 \\
n = 3539 \\
(n)(\hat{p}) = (3539)(0.560) \\
= 1981.84 \\
= 1982
\]

Answer:

Using TI-84 ➔ STAT ➔ TESTS ➔ A: 1-PropZInt

\[
= (0.544, 0.576)
\]