Statistics Workshop 5

Probability

What is Probability? Probability can be described as a value between 0 and 1 which represents an estimation of the likelihood that a certain event will occur.

Trial- A single attempt or evaluation of a random phenomenon.

Outcome- The observable result for a particular instance of a trial.

Event- A subset of the sample space.

Sample Space- The set of all possible outcomes (must sum to 1).

Independence- Two events are independent if the probable outcome of one event does not influence the probable outcome of another event.

Dependence- Two events are dependent if the outcome of one event exerts an influence over the outcome of another event.

Disjoint (aka mutually exclusive) - Events are disjoint when they share no common outcomes. In other words, one or the other may occur, but not both.

Formulas:

- The probability of a single simple event :
  \[ P(A) = \frac{\text{count of outcomes in } A}{\text{count of all possible outcomes}} \]

- The probability that an event does not occur (Complement Rule):
  \[ P(A) = 1 - P(A^c) \]

- The probability of events A or B (disjoint events):
  \[ P(A \text{ or } B) = P(A) + P(B) \]

- The probability of events A and B (General Multiplication Rule for independent events):
  \[ P(A \text{ and } B) = P(A) \times P(B) \]

- General Addition Rule- for “or” events (may be used for non-disjoint events):
  \[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
• General Multiplication Rule- for “and” events (can be used for dependent and independent events):
  \[ P(A \text{ and } B) = P(A) \times P(B|A) \]

• Conditional Probability Rule:
  \[ P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \]

**Example 1:** Finding the probability of a single simple event: \( P(A) = \frac{\text{count of outcomes in } A}{\text{count of all possible outcomes}} \)

<table>
<thead>
<tr>
<th>Sum</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>( \frac{1}{36} )</td>
<td>( \frac{2}{36} )</td>
<td>( \frac{3}{36} )</td>
<td>( \frac{4}{36} )</td>
<td>( \frac{5}{36} )</td>
<td>( \frac{6}{36} )</td>
<td>( \frac{5}{36} )</td>
<td>( \frac{4}{36} )</td>
<td>( \frac{3}{36} )</td>
<td>( \frac{2}{36} )</td>
<td>( \frac{1}{36} )</td>
</tr>
</tbody>
</table>

a.) Find \( P(\text{odd sum}) \)

Answer: \( P(3 \text{ or } 5 \text{ or } 7 \text{ or } 9 \text{ or } 11) = \frac{\frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{5}{36} + \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{18}{36} + \frac{1}{2}}{2} \)

b.) Find \( P(\text{sum at most 4}) \)

Answer: \( P(2 \text{ or } 3 \text{ or } 4) = \frac{\frac{1}{36} + \frac{2}{36} + \frac{1}{36} + \frac{6}{36} + \frac{1}{6}}{\frac{1}{6}} = \frac{\frac{18}{36} + \frac{1}{2}}{2} \)
Example 2: Finding the probability of events A or B: \( P(A \text{ or } B) = P(A) + P(B) \)
Applying the Complement Rule: \( P(A) = 1 - P(A^c) \)

<table>
<thead>
<tr>
<th>Car Color</th>
<th>Red</th>
<th>Blue</th>
<th>White</th>
<th>Black</th>
<th>Silver</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.25</td>
<td>.05</td>
<td>.25</td>
<td>.20</td>
<td>.10</td>
<td>.15</td>
</tr>
</tbody>
</table>

What is the probability that a randomly selected car is neither blue nor red?

Answer:

\[
1 - P(\text{Blue or Red}) = 1 - (0.05 + 0.25) = 1 - 0.30 = 0.70, \text{ or } 70\%
\]

Example 3: Using the General Multiplication Rule for independent events: \( P(A \text{ and } B) = P(A) \times P(B) \)

A telemarketing agency that sells vacation packages purchases phone lists of potential customers and installs them on its automated dialing system. Twenty percent of the numbers dialed in which a potential customer answers the phone result in a sale. What is the probability that the next three customers dialed do not make a purchase?

Answer:

\[
P(\text{sale}) = 0.20 \quad P(\text{no sale}) = 1 - P(\text{Sale})
\]

\[
= 1 - 0.20
\]

\[
= 0.80
\]

\[
P(\text{no sale and no sale and no sale}) = (0.80)(0.80)(0.80), \text{ which can also be written as } (0.80)^3
\]

\[
= 0.512, \text{ or } 51.2\%
\]
Example 4: The General Addition Rule for “or” events: \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \)

At a children’s birthday party, the probability that a guest will eat cake is .85, the probability that a guest will eat ice cream is .59, and the probability that a guest will eat both cake and ice cream is .52. What is the probability of a person eating cake or eating ice cream?

Answer:

We use the General Addition Rule for the events “eating cake” and “eating ice cream”, because the events are not disjoint. The General Addition formula allows us to eliminate the overlap (i.e. to avoid counting those who eat cake and ice cream twice).

\[
P(\text{cake or ice cream}) = P(\text{cake}) + P(\text{ice cream}) - P(\text{cake and ice cream})
\]

\[
= P(.85) + P(.59) - (.52)
\]

\[
= .92
\]

Example 5: General Multiplication Rule for “and” events: \( P(A \text{ and } B) = P(A) \times P(B | A) \)

A large grocery store chain reviewed its data from consumer purchases and found that 68% of its customers’ purchases included milk. Of these customers, 27% also bought breakfast cereal. If a customer is selected at random, find the probability that the customer has included both milk and cereal in the purchase.

Answer:

The formula \( P(A \text{ and } B) = P(A) \times P(B | A) \) can be interpreted as “the probability of A and B is equal to the probability of A times the probability of B, given A”

\[
P(A \text{ and } B) = P(.68) \times P(.27)
\]

\[
= .184, \text{ or } 18.4\%
\]
**Example 6:** Conditional Probability Rule: $\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$

At a local high school, students were asked if they favored a digital yearbook instead of a printed copy. **Thirty seven percent** of the student population favored the digital yearbook. Seniors made up **twenty one percent** of the student population. **Five percent** of the students were seniors who favored the digital yearbook. Find the probability that a randomly selected student favored the digital yearbook if the student was a senior.

**Answer:**

Seniors who favored the digital yearbook = $\Pr(A \text{ and } B) = .05$

Students who were seniors = $\Pr(A) = .21$

Therefore, the probability that a randomly selected student favored the digital yearbook if the student was a senior is $\frac{.05}{.21} = .238$, or 23.8%