Statistics Workshop 2

The Normal Model and Standard Deviation

**Median**- The middle value; associated with quartiles.

**Mean**- Obtained by taking the sum of all data values and dividing by the count; associated with the standard deviation.

**Variance**- Used to evaluate the spread of data.

**Variance Formula:**

\[
S^2 = \frac{\sum (x-\bar{x})^2}{n-1}
\]

**Standard Deviation**- Used to measure the spread of quantitative data in a distribution.

**Standard Deviation Formula:**

\[
S = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}
\]

To find the standard deviation using a TI 83/TI 84:

STAT→EDIT→Enter data into L1 (enter the numbers one at a time and press “enter” after each one)→STAT→CALC→1:1-VarStats (press enter 2x)

Summary statistics will display, and the standard deviation will appear next to the symbol \( S_x \).

**Q: How does the standard deviation relate to the variance?**

**A: The standard deviation is the square root of the variance. For example, if the variance is 9, the standard deviation will be 3.**
Example 1: I.Q. scores /the Normal Model

Set up a Normal Distribution (Bell curve), using a mean of 100 and standard deviation of 15.

The Empirical Rule (68-95-99.7 rule) - In a normal model, we can expect:

- 68% of the data values in a distribution to fall within ± 1 standard deviation of the mean.
- 95% of the data values to fall within ± 2 standard deviations of the mean.
- 99.7% of the data values to fall within ± 3 standard deviations of the mean.

Example 2: Using the normal distribution from Example 1, sketch and label the model with respect to the Empirical Rule.
**Z-score** - Used to explain how a single data point relates to the mean (average) of a set of data. In other words, a z-score tells us the number of standard deviations above/below the mean a particular data point happens to be.

**Z-score formula:**

\[ Z = \frac{x - \bar{x}}{s} \]

**Example 3:**

Using the mean and standard deviation from Example 1, find the z-score for an I.Q. of 123.

Given:

- mean \((\bar{x}) = 100\)
- standard deviation \((s) = 15\)
- data point \((x) = 123\)

\[ Z = \frac{x - \bar{x}}{s} = \frac{123 - 100}{15} = 1.53 \]
Z-scores can be used to determine the percentile for a particular region of a normal distribution.

Example 4: Normal Models- using z-scores to locate percentages.

What percent of a standard normal model is found in each region? Be sure to draw a picture.

a.  \( z > 1.25 \)
   
   To solve using TI 83/TI 84: 2\(^{nd}\) \(\rightarrow\) DIST \(\rightarrow\) 2: Normcdf(1.25, 100) = .106, or 10.6%

b.  \( z < 2.0 \)
   
   To solve using TI 83/TI 84: 2\(^{nd}\) \(\rightarrow\) DIST \(\rightarrow\) 2: Normcdf(-100, 2.0) = .977, or 97.7%

c.  \(-.75 < z < 1.6 \)
   
   To solve using TI 82/TI 84: 2\(^{nd}\) \(\rightarrow\) DIST \(\rightarrow\) 2: Normcdf(-.75, 1.6) = .719, or 71.9%

d.  \(|z| > 0.5 \)
   
   To solve using TI 83/TI 84: 2\(^{ndd}\) \(\rightarrow\) DIST \(\rightarrow\) 2:Normcdf(.5, 100) = .308 \times 2 = .617, or 61.7%
Example 5: Normal Model- using percentages to locate z-scores.

In a standard normal model, what value(s) of Z cut(s) off the region described?

a. **The highest 15%?**
   To solve using TI 83/TI 84: 2^{nd}→DIST→3:InvNorm(.85) = **1.04**

b. **The lowest 20%?**
   To solve using TI 83/TI 84: 2^{nd}→DIST→3:InvNorm(.20) = **-.84**

c. **The middle 80%?**
   To solve using TI 83/TI 84: 2^{nd}→DIST→3:InvNorm(.10) = **-1.28, 1.28**
Example 6: Normal cattle (Intro Stats, 3rd Ed.)

Using $N(1152, 84)$, the normal model for weights of Angus steers, what percent of steers weigh:

a. Over 1250 lb?

\[ Z = \frac{x - \mu}{\sigma} = \frac{1250 - 1152}{84} = 1.17 \]

Using TI 83/TI 84: 2nd → DIST → Normcdf(1.17, 100) = .121, or 12.1%

b. Under 1200 lb?

\[ Z = \frac{1200 - 1152}{84} = .57 \]

Using TI 83/TI 84: 2nd → DIST → Normcdf(-100, .57) = .716, 71.66%

c. Between 1000 and 1100 lb?

\[ Z = \frac{1000 - 1152}{84} = -1.81 \]
\[ Z = \frac{1100 - 1152}{84} = -.619 \]

Using TI 83/TI 84: 2nd → DIST → Normcdf(-1.81, -.619) = .233, or 23.3%