**Math is just another language: Take charge! Be the Translator.**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Expression</th>
<th>Example</th>
<th>Mathematical Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td>added to</td>
<td>6 added to y</td>
<td>y + 6</td>
</tr>
<tr>
<td></td>
<td>more than</td>
<td>8 more than x</td>
<td>x + 8</td>
</tr>
<tr>
<td></td>
<td>the sum of</td>
<td>the sum of x and z</td>
<td>x + z</td>
</tr>
<tr>
<td></td>
<td>increased by</td>
<td>t increased by 9</td>
<td>t + 9</td>
</tr>
<tr>
<td></td>
<td>the total of</td>
<td>the total of 5 and y</td>
<td>5 + y</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td>minus</td>
<td>x minus 2</td>
<td>x − 2</td>
</tr>
<tr>
<td></td>
<td>less than</td>
<td>7 less than t</td>
<td>t − 7</td>
</tr>
<tr>
<td></td>
<td>decreased by</td>
<td>m decreased by 3</td>
<td>m − 3</td>
</tr>
<tr>
<td></td>
<td>the difference between</td>
<td>the difference between y and 4</td>
<td>y − 4</td>
</tr>
<tr>
<td></td>
<td>subtract ... from ...</td>
<td>subtract 9 from z</td>
<td>z − 9</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td>times</td>
<td>10 times t</td>
<td>10t</td>
</tr>
<tr>
<td></td>
<td>twice</td>
<td>twice w</td>
<td>2w</td>
</tr>
<tr>
<td></td>
<td>of</td>
<td>one-half of x</td>
<td>1/2 x</td>
</tr>
<tr>
<td></td>
<td>the product of</td>
<td>the product y and z</td>
<td>yz</td>
</tr>
<tr>
<td></td>
<td>multiplied by</td>
<td>y multiplied by 11</td>
<td>11y</td>
</tr>
<tr>
<td><strong>Division</strong></td>
<td>divided by</td>
<td>x divided by 12</td>
<td>x/12</td>
</tr>
<tr>
<td></td>
<td>the quotient of</td>
<td>the quotient of y and z</td>
<td>y/z</td>
</tr>
<tr>
<td></td>
<td>the ratio of</td>
<td>the ratio of t to 9</td>
<td>t/9</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>the square of</td>
<td>the square of x</td>
<td>x^2</td>
</tr>
<tr>
<td></td>
<td>the cube of</td>
<td>the cube of a</td>
<td>a^3</td>
</tr>
<tr>
<td><strong>Equality</strong></td>
<td>equals</td>
<td>=</td>
<td></td>
</tr>
<tr>
<td></td>
<td>is</td>
<td>=</td>
<td></td>
</tr>
<tr>
<td></td>
<td>is equal to</td>
<td>=</td>
<td></td>
</tr>
<tr>
<td></td>
<td>amounts to</td>
<td>=</td>
<td></td>
</tr>
<tr>
<td></td>
<td>represents</td>
<td>=</td>
<td></td>
</tr>
</tbody>
</table>
## Integer Relationships

<table>
<thead>
<tr>
<th></th>
<th>Definition</th>
<th>Examples</th>
<th>Algebraic expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Even Integers</strong></td>
<td>Integer divisible by 2</td>
<td>-8, 2, 144, 0</td>
<td>n</td>
</tr>
<tr>
<td><strong>Odd Integers</strong></td>
<td>Integer not divisible by 2</td>
<td>1, -7, 15, 139</td>
<td>n</td>
</tr>
<tr>
<td><strong>Consecutive Integers</strong></td>
<td>Integers that follow one another in order</td>
<td>17, 18, 19</td>
<td>n, n+1, n+2</td>
</tr>
<tr>
<td><strong>Consecutive Even Integers</strong></td>
<td>Even integers that follow one another in order by 2</td>
<td>14, 16, 18</td>
<td>n, n+2, n+4, where n is even</td>
</tr>
<tr>
<td><strong>Consecutive Odd Integers</strong></td>
<td>Odd integers that follow one another in order by 2</td>
<td>31, 33, 35</td>
<td>n, n+2, n+4, where n is odd*</td>
</tr>
</tbody>
</table>

**Hint:**

For 'The sum of two numbers is ...' type problems, think to yourself, "The first number is n, so the second number is the sum minus n"

**Example:** The sum of two numbers is 23.
- Let the first number = n
- The second number is 23 - n
Word Problems: Be the translator ...

Uniform Motion Problems
Type I (MAT 0028, Ch 5.1 / MAT 1033, Ch 2.2)

Rate x Time = Distance \( (rt = d) \)

General Strategy:
- Determine what you know, and what you need to find.
- If necessary, draw a picture to help visualize the problem.
- Write a numerical or variable expression for distance, rate, and time for each object.
- Enter them into a table format.

Example:
Two cars, one traveling 10 mph faster than the other, start at the same time from the same point and travel in opposite directions. In 3 hours, they are 300 miles apart. Find the rate of each car. (MAT 0028: Ch 5.5, Ex 2)

What do we know for each car?
1) Total Distance for both cars: 300 miles
2) Both cars traveled the same amount of time: 3 hours
3) Relationship of the cars' rates: One car is 10 mph faster than the other

What are you trying to find?
1) Average rate of each car
2) If we let the rate of the first car = \( r \), then the rate of the second car is \( r+10 \)

What relationship gives us insight to the equation we need to solve?
The distance traveled by both cars add up to 300,

Make a table:

<table>
<thead>
<tr>
<th>Rate</th>
<th>x</th>
<th>Time</th>
<th>=</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>First car</td>
<td>( r )</td>
<td>3</td>
<td></td>
<td>( 3r )</td>
</tr>
<tr>
<td>Second car</td>
<td>( r+10 )</td>
<td>3</td>
<td></td>
<td>( 3(r+10) )</td>
</tr>
</tbody>
</table>

Since the distance traveled by both cars add up to 300,

\[ 3r + 3(r + 10) = 300 \]

Type II (MAT 1033, Chapter 6.4)
Distance ÷ Rate = Time

General Strategy:
Determine what you know and what you need to find out. Draw a picture to help visualize the problem. Write a numerical or variable expression for distance, rate, and time for each object. Enter them into a table format.

Example:
A marketing executive traveled 810 miles on a corporate jet in the same amount of time it took to travel an additional 162 miles by helicopter. The rate of the jet was 360 mph greater than the rate of the helicopter. Find the rate of the jet. (MAT 1033: Ch 6.4, Ex 3)

What do we know?
1) Distance traveled by corporate jet: 810 miles
2) Distance traveled by helicopter: 162 miles
3) Rate of jet is 360 mph faster than helicopter
4) Both traveled same amount of time: Their travel times are equal

What are you trying to find?
1) If we let the rate of the helicopter = \( r \), then the rate of the jet is \( r+360 \)
2) Average rate of jet: \( r+360 \)

What relationship gives us insight to the equation we need to solve?
The jet and helicopter travel the same amount of time, so their times are equal

Make a table:

<table>
<thead>
<tr>
<th>Distance</th>
<th>÷</th>
<th>Rate</th>
<th>=</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Jet</td>
<td>810</td>
<td>( r+360 )</td>
<td>( \frac{810}{r+360} )</td>
<td></td>
</tr>
<tr>
<td>Helicopter</td>
<td>162</td>
<td>( r )</td>
<td>( \frac{162}{r} )</td>
<td></td>
</tr>
</tbody>
</table>

Since the jet and helicopter traveled the same amount of time, the times are equal.

\[ \frac{810}{r+360} = \frac{162}{r} \]
Rate of Work Problems

Rate of work $\times$ Time worked = Part Completed

General Strategy:

- Determine what you know, and what you need to find.
- For each person or machine, write a numerical or variable expression for the rate of work, time worked, and part of task completed.
- Enter them into a table format.

Example:

Jim can landscape a new lawn in 36 hours. Jane can do the job in 45 hours. How long would it take to landscape a lawn if both of them worked together?

What do we know?

1) The time it takes Jim to landscape 36 hours
2) The time it takes Jane to landscape 45 hours
3) Working together, they work the same amount of time
4) Working together, they get 1 job completed (part completed by Jim + part completed by Jane = 1 job)

What are you trying to find?

1) The time it takes for both working together $t$ hours

What relationship gives us insight to the equation we need to solve?

The parts of the job completed by each person adds up to 1 job

Make a table:

<table>
<thead>
<tr>
<th></th>
<th>Rate of work $\times$ Time Worked</th>
<th>Part Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim</td>
<td>$\frac{1}{36}$ $\times$ $t$</td>
<td>$\frac{t}{36}$</td>
</tr>
<tr>
<td>Jane</td>
<td>$\frac{1}{45}$ $\times$ $t$</td>
<td>$\frac{t}{45}$</td>
</tr>
</tbody>
</table>

Since the part completed by Jim + the part completed by Jane equals 1 job,

Solve $\frac{t}{36} + \frac{t}{45} = 1$

Value Mixture Problems

Type I

Amount $\times$ Cost per unit = Value of Ingredient

General Strategy:

- Determine what you know, and what you need to find.
- For each ingredient, write a numerical or variable expression for the cost of ingredient and value of amount used.
- For the mixture, do the same.
- Enter them into a table format.

Example:

To make a flour mix, a miller combined soybeans that cost $8.50 per bushel with wheat that cost $4.50 per bushel. How many bushels of each were used to make a mixture of 800 bushels that cost $5.50 per bushel?

(MAT 1033: p. 72, # 16)

What do we know?

1) Cost of soybeans per bushel $8.50$ per bushel
2) Cost of wheat per bushel $4.50$ per bushel
3) Total mixture was 800 bushels (amount)
4) Since we know that the total amount is 800 bushels, if we let $a$ be the amount of soybeans, then 800-$a$ is the amount of wheat

What are you trying to find?

1) Amount each of soybeans and amount of wheat

What relationship gives us insight to the equation we need to solve?

The value of soybeans and the value of wheat must add up to $4400$

Make a table:

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>$\times$</th>
<th>Cost/unit</th>
<th>=</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>soybeans</td>
<td>$a$</td>
<td></td>
<td>$8.5$</td>
<td></td>
<td>$8.5a$</td>
</tr>
<tr>
<td>wheat</td>
<td>800-$a$</td>
<td></td>
<td>$4.5$</td>
<td></td>
<td>$4.5(800-a)$</td>
</tr>
<tr>
<td>Mixture</td>
<td>800</td>
<td></td>
<td>$5.5$</td>
<td></td>
<td>$4400$</td>
</tr>
</tbody>
</table>

Since the value of soybeans and the value of wheat must total $4400$,

Solve $8.5a + 4(800 - a) = 4400$
Type II (Involving Percent)

Amount of solution (or alloy) x percent of concentration = Quantity of substance

General Strategy:
- Determine how the quantities of the substance in the individual amounts are related.
- Use the fact that the sum of the quantities of the substance being mixed is equal to the quantity of the substance after mixing.
- Enter them into a table format.

Example:
A butcher has some hamburger meat that is 22% fat and some that is 12% fat. How many pounds of each should be mixed to make 80 lbs of hamburger that is 18% fat? (MAT 1033: p. 79)

What do we know?
1) Hamburger meat 1 has 22% fat: % of concentration = 0.22
2) Hamburger meat 2 has 15% fat: % of concentration = 0.15
3) Want the mixture to be 18% fat: % of concentration of mixture = 0.18
4) Total amount of meat is 80 lbs

What are you trying to find?
1) Amount of each meat

What relationship gives us insight to the equation we need to solve?
The quantity of fat of the first hamburger + the quantity of fat of the second hamburger must equal the amount of fat in the mixture.

Make a table:

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>x</th>
<th>% of conc.</th>
<th>=</th>
<th>Quantity of Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamburger meat 1</td>
<td>A</td>
<td></td>
<td>0.22</td>
<td></td>
<td>0.22A</td>
</tr>
<tr>
<td>Hamburger meat 2</td>
<td>80-A</td>
<td></td>
<td>0.12</td>
<td></td>
<td>0.12(80-A)</td>
</tr>
<tr>
<td>Mixture</td>
<td>80</td>
<td></td>
<td>0.18</td>
<td></td>
<td>14.4</td>
</tr>
</tbody>
</table>

Since the quantity of fat of the first hamburger + the quantity of fat of the second hamburger must equal the amount of fat in the mixture,

**Solve:** \(0.22A + 0.12(80 - A) = 14.4\)
Word Problem Practice Problems

Uniform Motion Problem

1. The Ride for Health Bicycle Club has chosen a 36-mile course for this Saturday's ride. If the riders plan on averaging 12 mph while they are riding, and they have a 1-hour lunch break planned, how long will it take them to complete the trip?
   (Source: MAT 0028, p. 293, #118)
   Ans: 4 hours

2. Two joggers start at the same time from opposite ends of an 8-mile jogging trail and begin running toward each other. One jogger is running at a rate of 5 mph and the other jogger is running at a rate of 7 mph. How long, in minutes, after they start, will the two joggers meet?
   (Source: MAT 0028, p 293, #121)
   Ans: 40 min

3. Marcella walked from her home to a bicycle repair shop at a rate of 3.4 mph. After picking up her bike, she rode it home along the same route at a rate of 14 mph. Her total travel time was 1 hour. How far is Marcella's home from the shop?
   (Source: MAC 1033, p 74, #41)
   Ans: 2.8 miles
Value Mixture Problem
4. To make a flour mix, a miller combined soybeans that cost $8.50 per bushel with wheat that cost $4.50 per bushel. How many bushels of each were used to make a mixture of 800 bushels that cost $5.50 per bushel?
(Source: MAC 1033, p 72, #16)
Ans: 200 bushels of soybeans, 600 bushels of wheat

Percent Mixture Problem
5. How many pounds of a 20% copper alloy must be mixed with 600 lbs of a 30% aluminum alloy to make a 27.5% copper alloy?
(Source: MAC 1033, p 82, #36)
Ans: 200 lbs

Investment Problems
6. Two investments earn an annual income of $765. One investment earns an annual simple interest rate of 8.5%, and the other investment earns an annual simple interest rate of 10.2%. The total amount invested is $8000. How much is invested in each account?
(Source: MAC 1033, p 80, #14)
Ans: $3000 at 8.5% and $5000 at 10.2%
Rate of Work Problems

A mechanic requires 2 hours to repair a transmission, whereas an apprentice requires 6 hours to make the same repairs. The mechanic worked alone for 1 hour and then stopped. How long will it take the apprentice, working alone, to complete the repairs?
(Source: MAT 0028, p 613, #12)
Ans: 3 hours

Integer Relationship Problems

Find three consecutive even integers whose sum is negative eighteen.
(Source: MAT 0028, p 314, #18)
Ans: -8, -6, -4

A 14-yd fishing line is cut into two pieces. Three times the length of the longer piece is four times the length of the shorter piece. Find the length of each piece.
(Source: MAT 0028, p. 316, #38)
Ans: 6 yd, 8 yd