Factoring: A General Strategy

To factor a polynomial:

First, always look first for a common factor. If there is one, factor out the largest common factor.

Then look at the number of terms.

Two terms: Determine whether you have

1) a difference of two squares: \( x^2 - y^2 = (x + y)(x - y) \)
2) a sum of two squares: \( x^2 + y^2 \) Do not try to factor a sum of squares.
3) a difference of two cubes: \( x^3 - y^3 = (x - y)(x^2 + xy + y^2) \)
4) a sum of two cubes: \( x^3 + y^3 = (x + y)(x^2 - xy + y^2) \)

Three terms: Determine whether you have

1) a perfect trinomial square: \( x^2 + 2xy + y^2 = (x + y)^2 \)
   \( x^2 - 2xy + y^2 = (x - y)^2 \)
2) a trinomial in the form \( x^2 + bx + c \).
   Step 1: Find two integers whose product is \( c \) and whose sum is \( b \).
   Step 2: Write as factors: \( (x + ?)(x + ?) \)
3) a trinomial in the form \( ax^2 + bx + c \)
   Step 1: Find two integers whose product is \( ac \) and whose sum is \( b \).
   Step 2: Rewrite the trinomial as a four-term polynomial, with the middle term expressed by the two integers you’ve chosen.
   Step 3: Factor by grouping.

OR, as an alternate approach, try repeated combinations (trial and error).

Four terms: Try factoring by grouping:

Step 1: Group terms with two terms in each group.
Step 2: Factor within the groups.
Step 3: Factor the entire polynomial.

Solving equations by factoring:

Step 1: Rewrite the equation if necessary so that one side is 0.
Step 2: Factor the polynomial.
Step 3: Set each factor equal to 0.
Step 4: Solve each equation.